Supporting Proof Comprehension Through Unpacking the Framework of Mathematical Ideas: A Case Study of Math 486 Concepts of Secondary Mathematics

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Key Questions

Proof and proving arise in most college mathematics courses, yet student difficulties with many aspects of proof persist (Selden&Selden 2008). Proof comprehension is a relatively open area of research (Mejia-Ramos&Inglis 2009). We study a scaffold intended to help students connect pieces of mathematics and improve their self-efficacy in communicating and comprehending proof. Our study is based on the questions:

How does an explicit emphasis on "**key ideas**" and the **logical architectures** of proofs influence:

1) students' ability to communicate and comprehend mathematics?
2) students' self-efficacy in communication and reading?

About the Study

We examine the idea that articulation of the practice of constructing and writing mathematical ideas is critical to producing and comprehending proofs. In Winter 2010, 30 students enrolled in MATH486 (Concepts of Secondary Mathematics). As an explicit introduction to the notion of key ideas, the students examined a proof of the Fundamental Theorem of Algebra, rewriting the main propositions of the proof in informal language, and explaining the connection between the informal and formal. To emphasize the logical architecture of proofs, students analysed a sequence of results regarding functions and how they might be used in proving a particular set of propositions, using an intervention similar to one studied by Weber (2006).

Sources of data analyzed include

- exam questions
- presentation write-ups
- survey
- in-class discussion sheets

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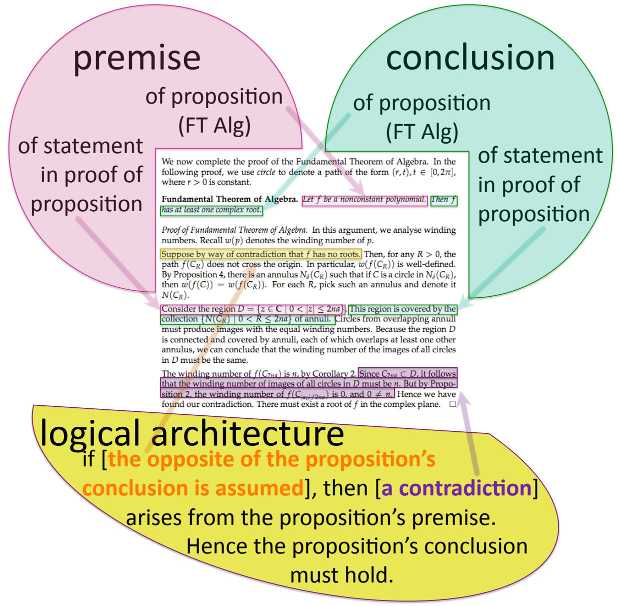
Elements of Mathematical Communication



Key Idea: essence of an idea or proof which can be rendered into mathematically rigorous argument or formal structure (Raman 2002)

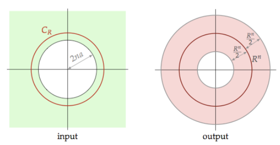


Figure 1. Visual schematic for Proposition 1. If C_R is contained in the shaded input region, then its image is contained in the shaded output region.

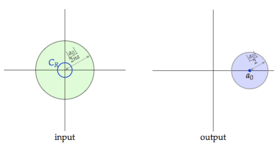


Figure 2. Visual schematic for Proposition 2. If C_R is contained in the shaded input region, then its image is contained in the shaded output region.

Conversational Mode

↑

notational facility, familiarity with formal mode

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Formal Mode

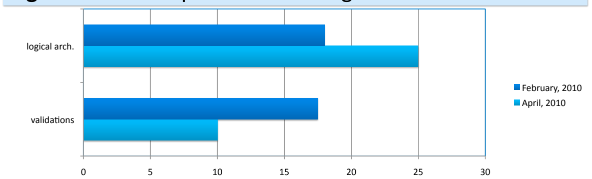
Proposition 1. *If $R \ge 2na$, then*

$$f(C_R) \subset \left\{ z \in \mathbb{C} \,\middle|\, \frac{1}{2}R^n \le |z| \le \frac{3}{2}R^n \right\}.$$

Proposition 2. *When $R \le \frac{|a_0|}{2na}$, then $f(C_R)$ is contained in a disk of radius $\frac{|a_0|}{2}$ centered at a_0.*

Preliminary Data

Data (**N=30**) from exam questions concerning:
validations: monitors for premises in validating conclusion
logical arch.: comprehension of logical architecture



implied hypothesis for further work:
comprehension of logical architecture is independent of validation skills

Representative responses to the question, **"How has this class shaped your approach to proofs and mathematical ideas?"**

- *It has taught me to look more in depth at mathematical ideas and has sparked a curiosity about what is behind things we take for granted in math.*
- *I've never liked proofs before. Actually, I hated them. This class has changed that because I feel like everything used to prove things is relevant. Even proving the "little" proofs is fun to me now (lemmas, etc.).*
- *This class has been very empowering because it shows that complicated proofs ideas can be made approachable.*

Implications

- Validation of mathematical statements and detecting logical framework may be independent learning constructs. In particular, **detecting logical architecture might not depend on validation skills.**
- An emphasis on key ideas and logical architecture may enhance proof comprehension and self-efficacy but not necessarily the ability to validate isolated mathematical statements.

References

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